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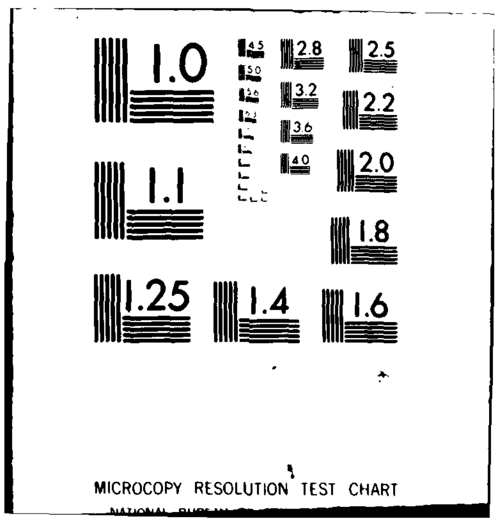
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Under Contract DAAG29 78 C 0026 for the period December 1, 1977 - November 30, 1980, seventeen reports ~~were completed or are in progress. These~~ are described.

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TIME-DEPENDENT MATHEMATICAL PROGRAMS

FINAL REPORT

B. CURTIS EAVES

DECEMBER 1, 1977 - NOVEMBER 30, 1980

U. S. ARO - DURHAM

DAAG-29-78-C-0026

DEPARTMENT OF OPERATIONS RESEARCH
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

FINAL REPORT

Under Contract DAAG-29-78-C-0026 for the period December 1, 1977 - November 30, 1980:

1. Seventeen reports were completed or are in progress. These are described on attached sheets.
2. Aside from the principal investigator, scientific personnel supported under the contract were: R. Asmuth, C. Engles, R. Freund, P. Brooks, J. Fonlupt and A. Rosenberg. The first three received the degree of Ph.D. of Operations Research during this period.

1. R. Asmuth, "Traffic Network Equilibria", January 1978.

A model of traffic flow on a road network is considered. For each ordered pair of nodes there is a demand function which expresses travel demand between the two nodes as a function of travel times on the network. Each road (arc) has a delay function which expresses travel time on that arc as a function of total traffic flow. The objective is to show how an equilibrium of travel times, flows, and demands may be computed under conditions which are simple, general, and plausible.

To solve the network problems we develop techniques for solving the stationary point problem. These techniques for the stationary point problem are the best of which we are aware.

2. B. C. Eaves, "A Locally Quadratically Convergent Algorithm for Computing Stationary Points", May 1978.

Stationary points for a system with a convex polyhedral set and a continuously differentiable function with positive definite derivatives are computed by iteratively solving the linearized problem. The procedure is shown to be a mixing of a finite number of Newton methods and to have a local convergence rate which is quadratic. The stationary point problem is of the type arising from the PIES energy model.

3. B. C. Eaves, "A View of Complementary Pivot Theory (or Solving Equations with Homotopies)", Functional Differential Equations and Approximation of Fixed Points, Eds: H-O. Peitgen and H-O Walther, Springer-Verlag, N.Y., 1979; also, Constructive Approaches to Mathematical Models", Eds: Coffman and Fix, Academic Press, 1979.

A brief, valid, and painless view of the equation solving computational method variously known as complementary pivot theory and/or fixed point methods is given. The view begins in Section 2 with a bit of history and some of the successes of the method. In Section 3 the unique convergence proof of the method is elucidated with a riddle on ghosts. In Section 4 the general approach for solving equations with complementary pivot theory is encapsulated in the "homotopy principle." In Section 5 a simple example is used to illustrate both the convergence proof and the "homotopy principle." Rudiments of the general theory are stated and the "main theorem" is exhibited in an example in Section 6. Two representative complementary pivot algorithms are presented vis-a-vis the "homotopy principle" and "main theorem" in Section 7. Finally, in Section 9 the principal difficulty of the method is discussed and some of the studies for dealing with this difficulty are mentioned.

4. C. R. Engles, "Economic Equilibrium Under Deformation of the Economy", Analysis and Computation of Fixed Points, Ed: Stephen M. Robinson, Academic Press, 1980.

A philosophical problem arises when one attempts to predict a competitive economy's response to a fundamental change in its structure with the aid of a competitive equilibrium model. Unless the model is known to admit unique solutions, there is little basis for assuming that the computed equilibrium will be attained, even though the model accurately describes the economy's structure and the behavior of its agents. If, however, one is able to arrive at the new model by continuously deforming the old one, then the two versions generally admit solutions which are connected by a path of equilibria arising from

the continuum of intermediate economies. By ascribing a suitable dynamic interpretation to the deformation, one obtains a rationale for expecting the path-connected solutions to be mutually attained.

The description of economic deformations and the computation of equilibrium paths is the central theme of this study. A general mathematical framework for modeling economies under deformation is developed by expanding Herbert Scarf's original activity analysis formulation to include uncountable unit activity sets, unbounded multi-valued demand correspondences, and tax and revenue systems similar to those introduced by John Shoven and John Whalley. Deformations of virtually all economic constructs are allowed in this general model.

The computation of equilibrium paths is accomplished by a simplicial pivot algorithm designed along the lines of the homotopy-type fixed point techniques pioneered by Curtis Eaves. The dimension normally used to refine piecewise linear approximations now serves as the index of the economic deformation. To make this approach viable in practice, a new family of triangulations of Euclidean space is fashioned out of two conventional triangulations invented by Michael Todd. The geometry of these triangulations can be dynamically altered by the algorithm as it attempts to maintain uniform approximation error along the equilibrium path.

The economic model and computational algorithm are translated into a set of computer routines which generate explicit numerical approximations to equilibrium paths for a variety of examples. Due to the vast amount of information embodied in an equilibrium path, problems of this type require a great deal of computational effort. A detailed analysis of the behavior of the algorithm on a series of test problems is presented in the final chapter.

5. P. S. Brooks, "Infinite Retrogression in the Eaves-Saigal Algorithm", Mathematical Programming 19 (1980) 313-327.

For certain functions $F : \mathbb{R}^n \times [0,1] \to \mathbb{R}^n$, the Eaves-Saigal algorithm computes a path $p = (p_1, p_2) : [0, +\infty) \to F^{-1}(0) \cap \mathbb{R}^n \times (0,1]$, such that $(p_1(s), p_2(s)) \to (z,0)$ as $s \to +\infty$. It is shown that even when $F(\cdot,0)$ is of a class C^∞ and has a unique zero, $p_2(s)$ may not decrease monotonically to 0 on $[s_0,+\infty)$ for any s_0.

6. B. C. Eaves and R. M. Freund, "Inscribing and Circumscribing Convex Polyhedra", March 1979. (Continued in No. 11).

Let $\mathscr{X}$ and $\mathscr{Y}$ be closed polyhedral convex sets, bounded or not, in R^n. For certain representations of $\mathscr{X}$ and $\mathscr{Y}$ it is shown that the task of finding the smallest scale of $\mathscr{X}$ for which some translate contains $\mathscr{Y}$ can be resolved with linear programming.

7. B. C. Eaves and C. E. Lemke, "Equivalence of LCP and PLS", December 1979, to appear in Mathematics of Operations Research.

The linear complementarity problem and the piecewise linear system are the two principal models in complementary pivot and fixed point theory. It is shown that the path following methods for solving each are comceptually equivalent.

8. R. M. Freund, "A Constructive Proof of the Borsuk-Ulam Antipodal Point Theorem", May 1979.

A proof of the Borsuk-Ulam Antipodal Point Theorem is presented by means of a constructive algorithm that computes an approximate solution by means of a simplicial subdivision and integer labels.

9. R. M. Freund, "Variable-Dimension Complexes with Applications", June 1980.

In the past few years, researchers in fixed-point methods have developed a number of variable-dimension simplicial algorithms. These algorithms are shown to be specific realizations of pivoting methods on a $\vee$-complex. The concept of a $\vee$-complex is also used to give new and constructive proofs of a variety of known theorems in combinatorial topology and mathematical programming. Finally, $\vee$-complexes give rise to new theorems in complementarity theory and combinatorial topology, including a generalization of the Sperner Lemma, a covering theorem on the simplex, and a new combinatorial lemma on the n-cube.

10. R. M. Freund and M. J. Todd, "A Constructive Proof of Tucker's Combinatorial Lemma", June 1980, to appear in Journal of Combinatorial Theory, Series A.

Tucker's combinatorial lemma is concerned with certain labellings of the vertices of a triangulation of the n-ball. It can be used as a basis for the proof of antipodal-point theorems in the same way that Sperner's lemma yields Brouwer's theorem. Here we give a constructive proof, which thereby yields algorithms for antipodal-point problems. Our method is based on an algorithm of Reiser.

11. B. C. Eaves and R. M. Freund, "Optimal Scaling of Balls and Polyhedra", August 1980, to appear in Mathematical Programming.

By a cell we mean either a nonempty closed polyhedral convex set or a nonempty closed solid ball. Out concern is with solving as linear or convex quadratic programs special cases of the optimal containment problem and the optimal meet problem. The optimal containment problem is that of finding the smallest scale of a collection of cells for which

some translate covers a given collection of cells, whereas the optimal meet problem is that of finding the smallest scale of a collection of cells for which some translate meets each of a set of collection of cells.

12. B. C. Eaves and U. Rothblum, "Computation of Fixed Points in Ordered Fields with Application to an Invariant Curve Theorem" (in progress)

It is shown that the homotopy principle can be applied over an arbitrary ordered field, in particular, to the Hilbert field $R(\omega)$ where ω is an infinitesimal. The observation permits one to compute invariant curves of certain operators which in turn enables one to optimize certain stochastic models.

13. B. C. Eaves, "LU Factorization and Pivoting for Solving Equations with PL Homotopies" (in progress)

Our concern here is in solving a sequence of systems of form $Bx = b$ with $x \geq 0$ where each $n \times n$ matrix B differs from its predecessor in one column only. We shall undertake this task by following the scheme of Bartels, Golub, and Saunders which is based on the LU factorization of a matrix; which has been shown to be numerically stable. Emphasis is on specializing the coding for path following algorithms.

14. B. C. Eaves, "A Course in Triangulations for Solving Equations with PL Homotopies" (in progress)

This is a comprehensive development of triangulations as used in complementary pivot and fixed point methods. Many improvements have

been made in the representation and replacement rules for movement in triangulations.

15. B. C. Eaves and J. Yorke, "Surface Area to Volume and Average Directional Density of Tilings" (in progress)

Given a tiling of R^n by a figure C, it is shown that the average directional density of the tiling can be computed by dividing the total surface area of C by the volume of C. A number of applications of this principle are indicated.

16. B. C. Eaves and A. F. Veinott, Jr., "Decomposition of and Policy Improvements in Stopping, Negative and Positive Markov Decision Chains" (in progress)

Howard's policy improvement is adapted for solving larger classes of the finite state, finite action, infinite horizon Markovian decision chains wherein the criterion is the weighted sum of immediate rewards. The cases we are able to optimize include the stopping, negative, and positive decision chains.

17. J. Fonlupt, "Tensor Product of Convex Sets and the Bilinear Programming Problem" (in progress)

This publication is divided into two main parts:

• In the first part the tensor product of two convex sets is defined, and the facial structures of the tensor products is studied. The tensor product of convex sets is examined as a special case.

• In the second part an algorithm for the bilinear programming problem based on the notion of tensor product is developed. The proposed method is compared with some existing methods.

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